Exercise 17

Find dy/dx by implicit differentiation.

$$\tan^{-1}(x^2y) = x + xy^2$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}[\tan^{-1}(x^2y)] = \frac{d}{dx}(x + xy^2)$$

$$\frac{1}{1+(x^2y)^2} \cdot \frac{d}{dx}(x^2y) = \frac{d}{dx}(x) + \frac{d}{dx}(xy^2)$$

$$\frac{1}{1+(x^2y)^2} \cdot \left\{\left[\frac{d}{dx}(x^2)\right] y + x^2 \left[\frac{d}{dx}(y)\right]\right\} = (1) + \left[\frac{d}{dx}(x)\right] y^2 + x \left[\frac{d}{dx}(y^2)\right]$$

$$\frac{1}{1+(x^2y)^2} \cdot \left[(2x)y + x^2(y')\right] = (1) + (1)y^2 + x\left[(2y) \cdot \frac{d}{dx}(y)\right]$$

$$\frac{2xy + x^2y'}{1+x^4y^2} = 1 + y^2 + 2xyy'$$

Solve for y'.

$$\frac{2xy}{1+x^4y^2} + \frac{x^2y'}{1+x^4y^2} = 1 + y^2 + 2xyy'$$

$$\left(\frac{x^2}{1+x^4y^2} - 2xy\right) y' = 1 + y^2 - \frac{2xy}{1+x^4y^2}$$

$$y' = \frac{1 + y^2 - \frac{2xy}{1+x^4y^2}}{\frac{x^2}{1+x^4y^2} - 2xy}$$

$$y' = \frac{\frac{(1+y^2)(1+x^4y^2)-2xy}{1+x^4y^2}}{\frac{x^2-2xy(1+x^4y^2)}{1+x^4y^2}}$$

$$y' = \frac{(1+y^2)(1+x^4y^2) - 2xy}{x^2 - 2xy(1+x^4y^2)}$$